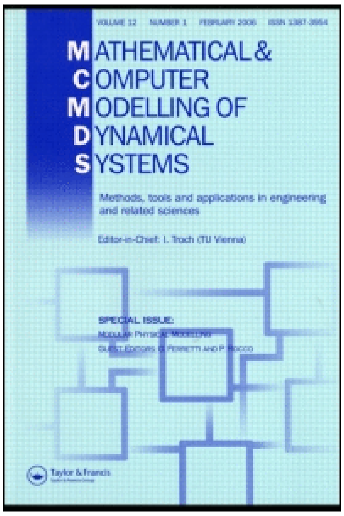

Mathematical and Computer Modelling of Dynamical Systems
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713682513

An explicit equation for the dynamics of a particle conveyed by a propagating wave

Minvydas Ragulskis [a]; Edita Sakyte [a]
[a] Department of Mathematical Research in Systems, Kaunas University of Technology, Kaunas, Lithuania

Online Publication Date: 01 August 2009

To cite this Article Ragulskis, Minvydas and Sakyte, Edita(2009)'An explicit equation for the dynamics of a particle conveyed by a propagating wave',Mathematical and Computer Modelling of Dynamical Systems,15:4,395 — 405

To link to this Article: DOI: 10.1080/13873950903063108

URL: http://dx.doi.org/10.1080/13873950903063108

Mathematical and Computer Modelling of Dynamical Systems
Vol. 15, No. 4, August 2009, 395–405

An explicit equation for the dynamics of a particle conveyed by a propagating wave

Minvydas Ragulskis* and Edita Sakyte

Department of Mathematical Research in Systems, Kaunas University of Technology, Kaunas, Lithuania

(*Received 27 November 2007; final version received 23 April 2009*)

An explicit governing equation of motion describing nonlinear dynamics of a particle conveyed by a propagating surface wave is deduced. A dynamic equilibrium is constructed at the contact point of the particle and the surface. The mathematical model of the system is constructed in such a way that it involves dynamically shifted coordinates around the contact point. Such an approach yields an explicit nonlinear differential equation. Coexisting attractors and their basin boundaries can be analysed in the general case. Special computational techniques are developed for numerical integration of such differential equations with dynamically shifted coordinates. Attractor control strategy based on small external impulses is proposed when stable equilibrium points and a limit cycle coexist. Such control strategies can dramatically increase the effectiveness of operation and can be applicable in different areas of engineering where positioning or conveyance is performed by means of propagating surface waves.

Keywords: propagating wave; nonlinear dynamics; explicit equation

1. Introduction

The conveyance of particles and bodies by propagating waves is an important scientific and engineering problem with numerous applications. Manipulation of bioparticles and gene expression profiling using travelling wave dielectrophoresis [1–3], segregation of particles in suspensions subject to ac electric fields [4], transport of sand particles and oil spills in coastal waters [5,6], powder transport by piezoelectrically excited ultrasonic waves [7,8] and transportation of thin films in biomedical applications [9] are just few examples of problems involving interaction between propagating waves and transported objects.

A basic model of a particle conveyed by a propagating wave comprises a mass particle in a gravitational field and a horizontal elastic conveyor. There have been numerous attempts to analyse such dynamical systems [10–13]. All these papers incorporate some sort of simplification. An exact governing equation of motion describing the conveyance of a mass particle by a propagating surface is derived in [9]. Unfortunately, this equation is implicit and cannot be exploited for analysis of equilibrium points. One of the main objectives of this article is to develop an explicit model of the system that would allow to perform a formal analysis of fixed points. That is very important for construction of basin boundaries whenever different attractors coexist.

*Corresponding author. Email: minvydas.ragulskis@ktu.lt

ISSN 1387-3954 print/ISSN 1744-5051 online

DOI: 10.1080/13873950903063108
http://www.informaworld.com

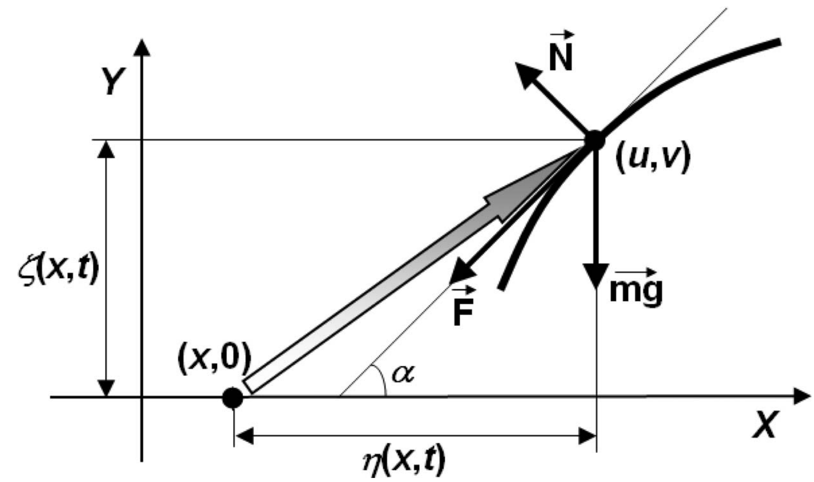

Figure 1. A geometric scheme of the dynamical system.

A 2D system is analysed for simplicity. It is assumed that the surface coincides with the x-axis in the state of equilibrium before the wave process took place (Figure 1). A point of the surface in the equilibrium state $(x, 0)$ is translated to coordinates (u, v) at time moment t. This translation is sensitive to time t and coordinate x:

$$\begin{aligned} u &= x + \eta(x, t) \\ v &= \zeta(x, t) \end{aligned} \tag{1}$$

where $\eta(x, t)$ and $\zeta(x, t)$ are predefined functions.

Kinematic relationships

$$\begin{aligned} \eta(x, t) &= a \sin(\omega t - kx) \\ \zeta(x, t) &= b \cos(\omega t - kx) \end{aligned} \tag{2}$$

in Equation (1) describe a retrograde elliptical motion of a travelling Rayleigh wave in an elastic media at the surface of the flat boundary [14] where ω is the angular frequency, k is the wave number, a and b are longitudinal and transverse amplitudes of oscillations. The ratio between amplitudes of the transverse and the longitudinal deformations depend on the Poisson ratio, but in usual elastic media it is quite normal that $b/a = 1.5$ [15].

Rayleigh waves are dispersive due to a dependence of the wave's speed on its wavelength. A typical example is Rayleigh waves in the Earth where waves with a higher frequency travel more slowly than those with a lower frequency. Rayleigh waves thus often appear spread out on seismograms recorded at distant earthquake-recording stations [16]. Surface acoustic waves (SAW) generated by SAW devices on rough anisotropic materials also experience considerable dispersion [17]. On the other hand, film waves generated on a surface of a finite liquid bed [9] can be characterised by a single-frequency wave component. Therefore we concentrate on a one single-frequency steady-state Rayleigh wave propagation and disregard dispersion.

It is assumed that a mass particle is in contact with the deformed surface at a point (u, v) at a time moment t. The condition that the particle is located on the surface leads to the following constraint:

$$v = \zeta(x,\ t) \tag{3}$$

where x is to be found from the following algebraic equality (in which u is given and x is unknown):

$$x + \eta(x, t) = u. \tag{4}$$

In other words, the instantaneous shape of the surface cannot be described by an explicit function. Thus the derivation of the model for such an apparently simple dynamical system faces considerable complications. An exact governing equation of motion describing the conveyance of a mass particle by a propagating surface is expressed in the following form [9]:

$$A_1(x, t) \cdot \ddot{u} + A_2(x, t) \cdot \dot{u} + A_3(x, t) + A_4(x, t) \cdot (\dot{u})^2 = 0 \tag{5}$$

where expressions $A_1(x, t)$, $A_2(x, t)$, $A_3(x, t)$ and $A_4(x, t)$ are available in the above-mentioned paper. Unfortunately Equation (5) is implicit – it contains two variables u and x, which are cross-linked by the relationship (4). Nevertheless, Equation (5) is a valuable product in the sense that the dynamics of a particle is considered on a surface in which instantaneous shape cannot be described explicitly (the problem of hidden variable x).

A straightforward numerical approach using standard time-integrating techniques to compute the particle motion in combination with the calculation of the surface wave can help to produce a partial solution; such a technique has been employed in [9]. One of the drawbacks of such an approach is that the numerical solution of the ordinary differential Equation (5) using a direct time-marching technique requires to solve Equation (4) at every time step (because the function describing a surface of the propagating Rayleigh wave is implicit). Moreover, the solution of Equation (4) is not straightforward. It incorporates an iterative numerical algorithm for the determination of the unknown x at given u and t. This algorithm must be executed at every time step and slows down the computational process considerably. Anyway, a partial solution can be generated using straightforward numerical integration techniques.

But problems occur when the global dynamics of the system are to be analysed. Fixed saddle points must be determined analytically before basin boundaries of attractors can be constructed. That is impossible when both transverse and longitudinal deflections are present in a Rayleigh surface wave and the dynamics of the system is described by an implicit governing equation of motion Equation (5). Therefore there exists a definite need for an explicit governing equation of motion describing dynamics of a particle on a surface of propagating wave in the general case. But if the differential equation of motion is constructed at the contact point (u, v), the implicit formulation of the problem is unavoidable [9]. Alternatively, one could try to construct the differential equation of motion at a point $(x, 0)$. The contact point (u, v) on the surface would return to the point $(x, 0)$ if the surface would instantaneously return to the condition before the wave process took place. Such analysis with dynamically shifted coordinates would complicate the structure of the mathematical model of the system – dynamics of the particle would be considered at a point x, not the contact point u. On the other hand such an approach would be advantageous – the governing equation would be explicit and formal analysis of system's attractors would be possible in the general case.

Finally it can be mentioned that a possibility to construct basin boundaries of attractors can help to construct efficient motion-control strategies. These strategies have a far-reaching potential of applications (not only for a particle conveyed by a propagating wave).

2. Construction of an explicit governing equation of motion

Though the instantaneous shape of the oscillating surface cannot be described by an explicit function, the tangent to the surface at the point (u, v) can be expressed as follows:

$$\tan\alpha = \frac{\zeta'_x(x,t)}{1+\eta'_x(x,t)}. \tag{6}$$

Instantaneous velocities of the surface's point (u, v) in the direction of x- and y-axis can be expressed as follows:

$$\begin{aligned} \dot{u}|_{x=const} &= \eta'_t(x,t) \\ \dot{v}|_{x=const} &= \zeta'_t(x,t) \end{aligned} \tag{7}$$

where dots denote derivatives by t.

When a mass particle slides on the surface, it does not necessarily move in contact with one point of the surface. Therefore x is no longer a constant. Differentiation of Equation (1) produces

$$\begin{aligned} \dot{u} &= \dot{x}\left(1+\eta'_x\right) + \eta'_t \\ \ddot{u} &= \ddot{x}\left(1+\eta'_x\right) + \dot{x}^2\eta''_{xx} + 2\dot{x}\eta''_{xt} + \eta''_{tt}. \end{aligned} \tag{8}$$

The condition that the mass particle continuously slides on the surface brings another constraint into force (the relative velocity in the normal direction to the surface at the contact point must be zero):

$$\tan\alpha = \frac{\dot{v}-\zeta'_t(x,t)}{\dot{u}-\eta'_t(x,t)}. \tag{9}$$

Equation (6) with Equation (9) in force yields

$$\dot{v} = \frac{\dot{u}-\eta'_t}{1+\eta'_x}\zeta'_x + \zeta'_t \tag{10}$$

which together with Equation (8) produces the following relationship

$$\dot{v} = \dot{x}\zeta'_x + \zeta'_t. \tag{11}$$

Differentiation of Equation (11) yields

$$\ddot{v} = \ddot{x}\zeta'_x + \dot{x}^2\zeta''_{xx} + 2\dot{x}\zeta''_{xt} + \zeta''_{tt}. \tag{12}$$

Then the relative sliding velocity of the particle on the surface v_{12} can be expressed as

$$\begin{aligned} v_{12} &= \left(\dot{u}-\eta'_t\right)\cos\alpha + \left(\dot{v}-\zeta'_t\right)\sin\alpha = \frac{\dot{x}}{\sqrt{1+\tan^2\alpha}}\left(1+\eta'_x+\zeta'_x\tan\alpha\right) \\ &= \dot{x}\sqrt{\left(1+\eta'_x\right)^2+\zeta'_x 2}. \end{aligned} \tag{13}$$

The condition of dynamic equilibrium leads to the following system of equations:

$$\begin{cases} m\ddot{u} + N\sin\alpha + F\cos\alpha = 0 \\ m\ddot{v} + mg + F\sin\alpha = N\cos\alpha \end{cases} \tag{14}$$

where m is the mass of the particle N is the reaction force at the contact point, g is the gravity acceleration and F is the friction force between the mass particle and the surface. The system of equations in Equation (14) is in force when $N > 0$. Otherwise the particle jumps off the oscillating surface.

It is assumed that the friction force is linear. Thus F can be expressed as

$$F = hv_{12} \tag{15}$$

where h is the coefficient of linear friction.

Finally, the governing equation of motion can be derived from Equation (14). Elementary transformations and substitutions lead to the following explicit differential equation:

$$B_1(x,t)\cdot\ddot{x}+B_2(x,t)\cdot\dot{x}+B_3(x,t)+B_4(x,t)\cdot(\dot{x})^2=0 \tag{16}$$

where

$$\begin{aligned}
B_1(x,t)&=m\left(1+\eta'_x+\frac{(\zeta'_x)^2}{1+\eta'_x}\right)\\
B_2(x,t)&=2m\left(\eta''_{xt}+\frac{\zeta'_x\zeta''_{xt}}{1+\eta'_x}\right)+h\left(1+\eta'_x+\frac{(\zeta'_x)^2}{1+\eta'_x}\right)\\
B_3(x,t)&=m\left(\eta''_{tt}+g\frac{\zeta'_x}{1+\eta'_x}+\frac{\zeta'_x\zeta''_{tt}}{1+\eta'_x}\right)\\
B_4(x,t)&=m\left(\eta''_{xx}+\frac{\zeta'_x\zeta''_{xx}}{1+\eta'_x}\right).
\end{aligned} \tag{17}$$

A major obstacle is eliminated and direct numerical time-marching techniques can be used for integration of Equation (16) – computation of u and v is straightforward if the coordinate x is given at time t (Equation (1)). Existence of stability of the dynamic equilibrium can be analysed explicitly.

But before proceeding with the analysis of dynamic equilibrium the following observation can be done. If kinematic relationships in Equation (2) are in force, the change of variables

$$z=\omega t-kx \tag{18}$$

transforms Equation (16) to the following autonomous form:

$$C_1(z)\cdot\ddot{z}+C_2(z)\cdot\dot{z}+C_3(z)+C_4(z)\cdot(\dot{z})^2=0 \tag{19}$$

where

$$\begin{aligned}
C_1(z)&=-\frac{m}{k}\left(1-ka\cos(z)+\frac{k^2b^2\sin^2(z)}{1-ka\cos(z)}\right)\\
C_2(z)&=\frac{h}{m}C_1(z)\\
C_3(z)&=\frac{\omega h}{k}\left(1-ka\cos(z)+\frac{k^2b^2\sin^2(z)}{1-ka\cos(z)}\right)+mg\frac{kb\sin(z)}{1-ka\cos(z)}\\
C_4(z)&=-m\left(a\sin(z)+\frac{kb^2\sin(z)\cos(z)}{1-ka\cos(z)}\right).
\end{aligned} \tag{20}$$

An important conclusion can be arrived at. Dynamics of a particle sliding on the surface of a propagating Rayleigh wave cannot be chaotic. This is because the governing equation of motion is a second-order autonomous ordinary differential equation with smooth parameter functions [18].

3. The dynamic equilibrium

Equation (10) (with Equation (2) in force) yields the dynamic equilibrium, which represents a motion of the particle on a slope of the propagating wave with the velocity of its propagation:

$$\begin{aligned} \ddot{u} &= 0, \\ \dot{u} &= \omega/k, \\ u &= \omega/k \cdot t - \psi \end{aligned} \tag{21}$$

where ψ is a constant. Then, it follows from Equation (6) that

$$x + a\sin(\omega t - kx) = \omega/k \cdot t - \psi. \tag{22}$$

The term $a\sin(\omega t - kx)$ is bounded, therefore Equation (22) will be in force when

$$x = \omega/k \cdot t - \theta \tag{23}$$

where θ is a constant satisfying the equality $-\theta + a\sin(k\theta) = -\psi$. Moreover, conditions of existence of the dynamic equilibrium are similar in terms of x or u:

$$\begin{aligned} \dot{x} &= \frac{\omega/k - \eta'_t}{1 + \eta'_x} = \omega/k, \\ \ddot{x} &= \frac{-\dot{x}^2\eta''_{xx} - 2\dot{x}\eta''_{xt} - \eta''_{tt}}{1 + \eta'_x} = 0. \end{aligned} \tag{24}$$

A numerical value of u can be determined straight from Equation (6) when the co-ordinate x is known.The only difference is that the phase θ does not coincide with the phase ψ, but this does not complicate the computational process. Governing equations of motion are solved in terms of x, not u. A numerical value of u can be determined straight from Equation (6) when the co-ordinate x is known.

Conditions of the dynamic equilibrium (Equations (23) and (24)) are carried into Equation (18) and then into Equation (17). Elementary transformations lead to the following equality:

$$(A - B)r^4 + 2Cr^3 + 2(A + 2D)r^2 + 2Cr + (A + B) = 0 \tag{25}$$

where $A = \frac{h\omega}{km}(1 + k^2a^2)$, $B = -\frac{2ha\omega}{m}$, $C = kgb$, $D = \frac{kh\omega}{m}(b^2 - a^2)$, $r = \tan\frac{k\theta}{2}$. It can be noted that Equation (25) is a fourth-order polynomial, so there exist four roots – some of which may be real numbers. Thus the unknown θ can be expressed as follows:

$$\theta_{1,2,3,4} = \frac{2}{k}\left(\arctan r_{1,2,3,4} + \pi n\right), \quad n = 0, \pm1, \pm2, \ldots \tag{26}$$

The number of different real roots is determined exploiting Sturm's theorem [19]. Results are presented in the system's parameter space in Figure 2. It can be noted that the dimensionless value $g = 1$ is used for further numerical investigations. Numbers of real roots are shown using a grey-scale colour scheme: 0, real roots in white areas; 2, real roots in grey areas and 4, real roots in black areas.

The number of roots, their stability and analytical description of boundaries between different regions in the parameter space is presented in detail in [9] at $a = 0$ only. It can be noted that the range of the amplitude b in Figure 2c and 2d is between 0 and 1.5. This is because at $a/k = 1$ (what in our case corresponds to $b = 1.5$) the shape of the propagating surface degenerates to a propagating cyclone. Naturally, there is no need to analyse a non-physical behaviour of the system.

A maximum of two real roots exist in Figure 2c and 2d. One of these roots is a stable attracting equilibrium point and another is an unstable saddle-type repelling point. Thus only

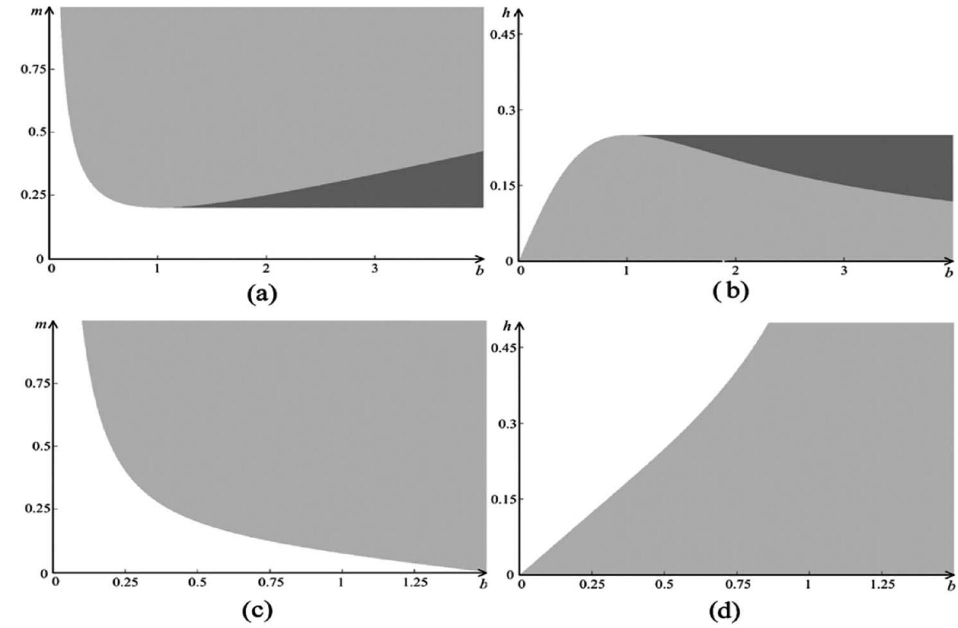

Figure 2. Numbers of real roots: (a) at $a = 0$, $h = 0.1$, $\omega = k = 1$; (b) at $a = 0$, $m = 0.5$, $\omega = k = 1$, (c) at $a = b/1.5$, $h = 0.1$, $\omega = k = 1$; (d) at $a = b/1.5$, $m = 0.5$, $\omega = k = 1$.

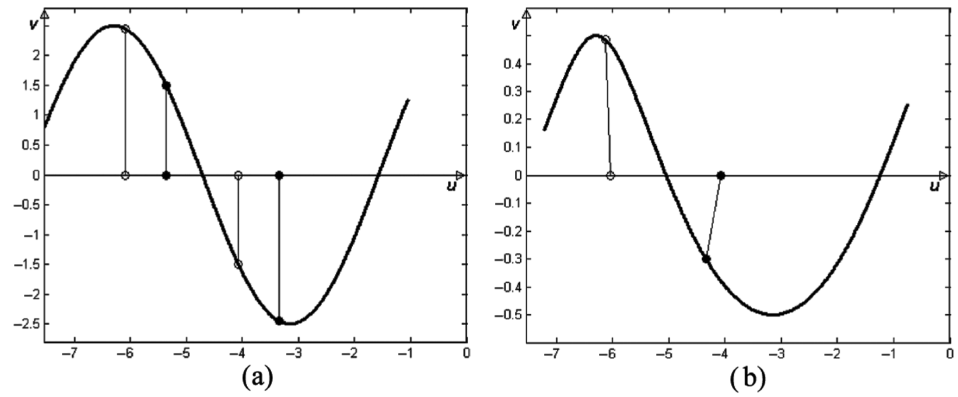

Figure 3. Equilibrium points; (a) at $b = 2.5$, $a = 0$, $m = 0.4$, $h = 0.1$, $\omega = k = 1$; (b) at $b = 0.5$, $a = b/1.5$, $m = 0.4$, $h = 0.1$, $\omega = k = 1$, black circles stand for stable equilibrium points, empty circles unstable saddle points.

one stable equilibrium point can exist on a slope of a propagating Rayleigh wave in the presented domain of parameters. This is a rather unexpected result keeping in mind that two stable equilibrium points can coexist at $\eta = 0$.

The location of equilibrium points on the slope of the propagating wave is presented in Figure 3a and 3b. Black circles denote stable equilibrium points and empty circles denote unstable saddle points. Equilibrium points on the surface are presented in the coordinate system $u0v$ and equilibrium points on the horizontal axis are presented in the coordinate system $x0v$. It can be noted that the location of equilibrium points do not coincide on the u-axis and on the x-axis because of the relationship described by Equation (4).

4. Numerical integration of the explicit equation

As mentioned earlier, the explicit governing equation is formulated in terms of x, not the coordinate of the contact point u. Nevertheless, it is much simpler to find u when x is known compared with the opposite (when the execution of iterative algorithms is necessary at every time step). Such computational advantage is illustrated in Figure 4. First, coordinates of the unstable saddle point are determined from Equation (26). Then coordinates of the same saddle point are calculated in the frame $(\omega t - ku), \dot{u}$ using the relationship in Equation (4). It can be noted that the saddle point is an unstable equilibrium point, so its coordinates in vertical axis ($\dot{x}$- and $\dot{u}$-axis) coincide (Equation (24)). Forward and reverse time-marching techniques are used to construct basin boundaries of attractors when partial solutions of Equation (16) are sought from the infinitesimal surrounding the saddle point. At the same time values of u and $\dot{u}$ are calculated from x and $\dot{x}$ at every time step using Equations (4) and (8) (both for forward and reverse integration in time). Solutions in terms of x are plotted as thin solid lines, solutions in terms of u are plotted as thick solid lines, all on the same frame. Such a numerical technique enables the visualisation of global properties of the analysed dynamical system in terms of u using the explicit governing equation of motion in terms of x.

The described computational technique is used to construct basin boundaries of the system's attractors (Figure 5). Solutions in terms of u (forward and reverse) only are visualised. It can be noted that two stable attractors can coexist – a stable equilibrium point and a stable limit cycle. Shaded regions in Figure 5 correspond to a basin (attracting set of initial conditions) of stable equilibrium points, whereas white region corresponds to a basin of the limit cycle. The phase plane in Figure 5 is periodic by 2π and can be visualised in cylindrical coordinates, but the plane representation is clearer.

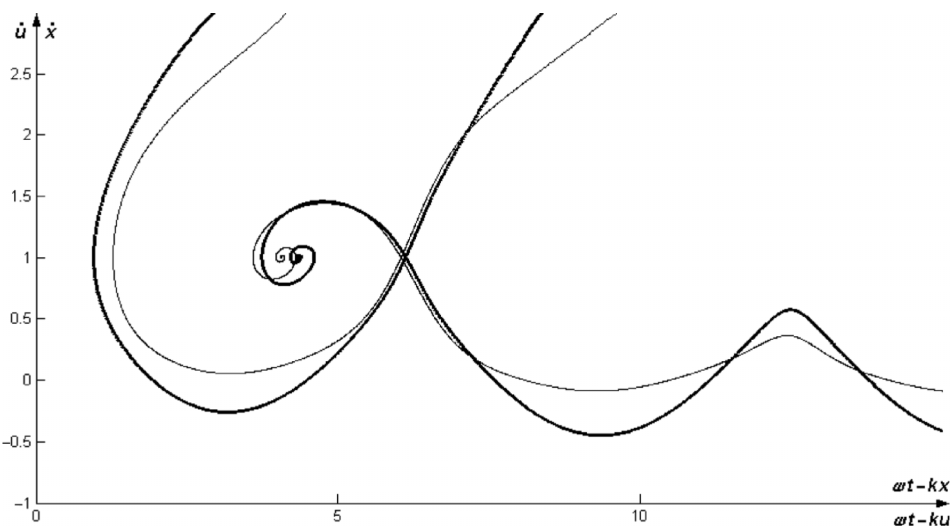

Figure 4. Basin boundaries constructed from a saddle-type repellor at $b = 0.5$, $a = b/1.5$, $m = 0.4$, $h = 0.1$, $\omega = k = 1$; thin solid lines stand for phase trajectories in terms of x (shifted coordinates); thick solid lines in terms of u.

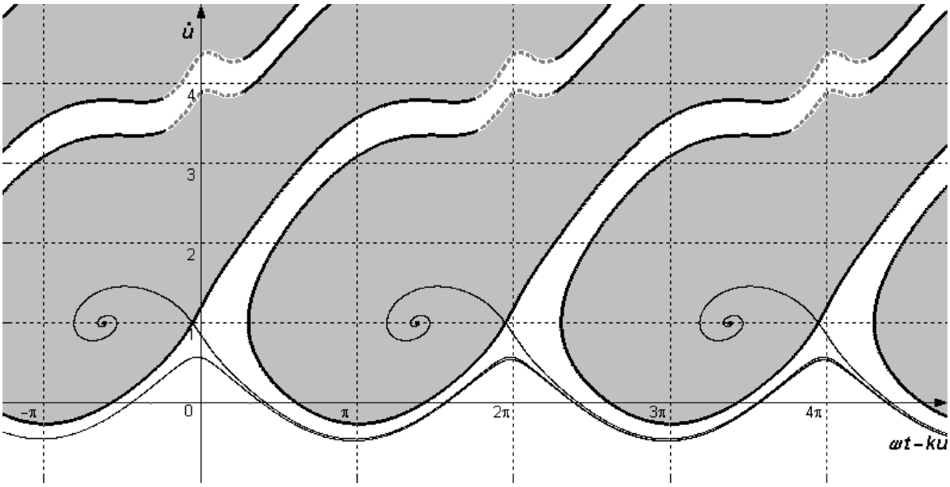

Figure 5. Basin boundaries at $b = 0.5$, $a = b/1.5$, $m = 0.4$, $h = 0.1$, $\omega = k = 1$; shaded regions illustrate the basin of attraction of stable equilibrium points.

Special attention should be paid to dashed line intervals on basin boundaries. Equation (16) describes a motion of a particle on the surface of a propagating wave. This governing equation of motion holds until the reaction force N in Equation (14) is positive. Whenever N gets equal or lower than 0, the particle loses contact with the surface and starts a free fly in a gravitational field until it hits the surface again. The object of this article is a nonlinear conveyance of a particle by a propagating wave when the conveyed particle slides on the oscillating surface. Therefore, the moment when the particle loses contact with the surface is detected and the trajectory is marked by a dashed line. It can be noted that such motions occur only at relatively high particle velocities (Figure 5).

Conveyance of a particle by a propagating Rayleigh wave is a nonlinear problem, so such effects as the coexistence of stable attractors should not be astonishing. Stable equilibrium point-type attractor in Figure 5 corresponds to a surf-type motion on a slope of a

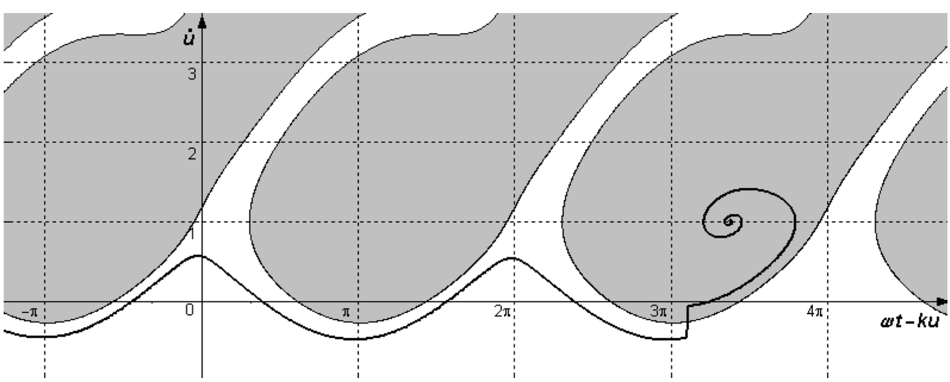

Figure 6. llustration of the attractor-control strategy: limit cycle is represented as a periodic trajectory in frame $(\omega t - ku; \dot{u})$; small external impulse kicks the trajectory to basin boundary of stable equilibrium point where the particle eventually settles down.

propagating wave; stable limit cycle corresponds to a motion with an average velocity much lower than the velocity of the propagating wave. Coexistence of attractors (a stable equilibrium point and a stable limit cycle) enables development of motion-control strategies based on small external impulses that can bring the system from the regime of motion with small average velocity into motion with the propagating wave's velocity [9]. Such attractor-control strategy is illustrated in Figure 6 where the particle first oscillates in the limit cycle and then a small external impulse kicks it to the basin of attraction of the stable focus point.

It can be noted that the above-mentioned control strategy can be implemented only when the stable equilibrium point and the stable limit cycle coexist. Thus it would be impossible to transport a sand particle with the velocity of the propagating wave by an acoustic surface Rayleigh wave. Nevertheless, such attractor-control strategies could be implemented for the transportation of biomedical objects on the surface of an undulation film [9], which is definitely an object of future research.

5. Concluding remarks

An explicit equation for the dynamics of a particle conveyed by a propagating wave is derived. This explicit equation is formulated at a dynamically shifted coordinate from the contact point between the particle and the surface. Analytical investigation of equilibrium points enables to construct basin boundaries of coexisting attractors characteristic to this dynamical system. Local dynamics is investigated using forward and reverse time-marching techniques, but they require special adaptations to cope with the dynamically shifted coordinate.

Direct time-marching techniques can also be used for the implicit equation describing the dynamics of a conveyed particle. But it is impossible to derive analytical conditions describing the existence and the stability of equilibrium points then, which prevents the construction of basin boundaries of coexisting attractors.

The characteristic shape of basin boundaries helped to develop an efficient attractor-control strategy for a particle conveyed by a propagating wave. Such motion-control strategies can be adapted for more complex dynamical systems. That is a definite object of future research.

References

[1] C.F. Chou, J.O. Tegenfeldt, O. Bakajin, S.S. Chan, E.C. Cox, N. Darnton, T. Duke, and R.H. Austin, *Electrodeless dielectrophoresis of single- and double-stranded DNA*, Biophys. J. 83 (2002), pp. 2170–2179.

[2] L. Cui and H. Korgan, *Design and fabrication of travelling wave dielectrophoresis structures*, J. Micromech. Microeng. 10 (2000), pp. 72–79.

[3] M.S. Talary, J.P.H. Burt, J.A. Tame, and R. Pethig, *Electromanipulation and separation of cells using travelling electric fields*, J. Phys. D Appl. Phys. 29 (1996), pp. 2198–2203.

[4] A.D. Dusiaud, B. Khusid, and B.A. Acrivos, *Particle segregation in suspensions subject to high-gradient ac electric fields*, J. Appl. Phys. 88 (2000), pp. 5463–5473.

[5] N.H. Wael and J.S. Ribberink, *Transport processes of uniform and mixed sands in oscillatory sheet flow*, Coastal Eng. 52, No. 9 (2005), pp. 745–770.

[6] S.D. Wang, Y.M. Shen, and Y.H. Zheng, *Two-dimensional numerical simulation for transport and fate of oil spills in seas*, Ocean Eng. 32 (2005), pp. 1556–1571.

[7] M. Mracek and J. Wallaschek, *A system for powder transport based on piezoelectrically excited ultrasonic progressive waves*, Mater. Chem. Phys. 90, No. 2–3 (2005), pp. 378–380.

[8] M. Ragulskis and M.A.F. Sanjuan, *Transport of particles by surface waves: a modification of the classical bouncer model*, New J. Phy. 10 (2008), article no. 083017.

[9] M. Ragulskis and K. Koizumi, *Applicability of attractor control techniques for a particle conveyed by a propagating wave*, J. Vib. Control 10 (2004), pp. 1057–1070.

[10] D. Benisti and D.F. Escande, *Explicit reduction of N-body dynamics to self-consistent particle-wave interaction*, Phys. Plasmas 4 (1997), pp. 1576–1581.

[11] Y. Elskens, D. Guyomarch, and M.C. Firpo, *Phase space dynamics and wave–particle interaction*, Phys. Mag. 20 (1998), pp. 193–203.

[12] L. Hui and Y. Tomita, *A numerical simulation of swirling flow pneumatic conveying in a horizontal pipeline*, Part. Sci. Technol. 18 (2000), pp. 275–292.

[13] A.M. Tokar and A.F. Ulitko, *Motion of material particle under the influence of elastic oscillations of a surface of a body*, Herald of Acad. Sci. Ukraine A7 (1984), pp. 46–49.

[14] J.D. Achenbach, *Wave Propagation in Elastic Solids*, Elsevier, New York, 1984.

[15] L.D. Landau and E.M. Lifschitz, *Theory of Elasticity*, Pergamon Press, Oxford, 1986.

[16] K. Aki and P.G. Richards, *Quantitative Seismology*, Freeman and Co., New York, 1980.

[17] C.M. Flannery and H. Kiedrowski, *Dispersion of surface acoustic waves on rough anisotropic materials*, IEEE Ultraso. Symp. 1 (2001), pp. 583–586.

[18] R.C. Hilborn, *Chaos and Nonlinear Dynamics*, Oxford University Press, Oxford, 2004.

[19] H. Dörrie, *Sturm's problem of the number of roots*, in *100 Great Problems of Elementary Mathematics: Their History and Solutions*, Dover, New York, 1965, pp. 112–116.