



Control of a dendritic neuron driven by a phase-independent stimulation



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ARTICLE INFO

Article history:

Received 17 November 2015

Accepted 16 January 2016

Keywords:

Dendritic neuron

Control pulse

Basin boundaries

ABSTRACT

A dendritic neuron model exhibits bistability under continuous weak stimulation – the oscillatory synchronized regime and the quiet regime coexist. Complex nonlinear dynamics is observed when the neuron undergoes not only phase-dependent continuous weak stimulation, but also when it is driven by an external phase-independent stimulation. In the latter case basin boundaries between the synchronized and the quiet regime become complex and fractal. Simple strategies based on control pulses are not sufficient in these circumstances, because it becomes difficult to predict the dynamics of the neuron after the application of the control pulse. Therefore, a new neural control method is proposed. Initially, a weak phase control strategy is applied until fractal basin boundaries evolve into a deterministic manifold. Consequently, a single control pulse is immediately applied and the neuron evolves into the calm state.

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1. Introduction

Numerous techniques based on nonlinear dynamics and chaos have been extensively used in the past years for modeling neuronal dynamics – either for continuous models [1] or for discrete models of neurons [2].

Synchronization processes of the brain is a hot topic of research during the last decades [3,4]. In several neurological diseases such as Parkinson's disease (PS) or essential tremor (ET), brain function is impaired by pathological synchronization processes [5,6]. It is shown in [7] that gap junctions play an important role in the synchronization of neuronal ensembles. Post-seizure additions of gap junctions could serve to prevent further escalations of the epileptic pathological activity of the brain. However,

the model in [7] predicts the existence of a critical tipping point after which additional gap junctions no longer suppress but strongly facilitate the escalation of epileptic seizures.

The importance of information transmission delays in the synchronization across small-world neuronal networks is investigated in [8]. It is demonstrated in [9] that increasing the coupling strength enhances synchronization of scale-free neuronal networks monotonously, whereas delay plays a more subtle role. It is shown in [9] that fine-tuned information transmission delays are vital for assuring optimally synchronized excitatory fronts on complex neuronal networks and, indeed, they should be seen as important as the coupling strength or the overall density of interneuronal connections. It is discussed in [10] that for both attractive and repulsive coupling, the delay always plays a subtle role in either promoting or impairing synchronization in scale-free neuronal networks.

It is shown in [11] that the scale-free nature of small-world networks, on one hand, facilitates signal

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transduction and thus temporal order in the system, whilst on the other hand, disrupts the internal spatial scale of the media thereby hindering the existence of coherent wave-like patterns. The existing experiments have shown that the spatiotemporal pattern and synchronization dynamics are very crucial, which are closely related to normal function and dysfunction of neuronal systems [12].

Electrical deep brain stimulation is one of the standard treatments of PS and ET [13,14] when a permanent high amplitude periodic pulse train suppresses neuronal firing [15,16]. However, the nonlinear dynamics of electrical deep brain stimulation is not yet fully understood [15]. It is well known that the therapeutic effect of the stimulation often decreases over time [17].

Novel pulse-based stimulation techniques are being developed which enable the selective desynchronization of pathologically synchronized processes in networks of phase oscillators as shown in Refs. [18–23]. Standard nonlinear dynamics and chaos methods for the analysis of neurodynamics can be applied [24]. Though generic phase oscillators can be used to approximate the dynamics of periodically active neurons [3,25–27], the pulse-based stimulation requires to take the dendritic dynamics of neurons into account [28–30]. Individual neurons are capable of exhibiting transient dynamics when exposed to electrical stimulation [31,32]. Incorporation of the dendritic dynamics into a generic phase oscillator model helps to describe the effective inertia of a neuron when the response of the neuron's state to perturbations is not instantaneous but exhibits smooth transient dynamics [28,33]. The dendritic dynamics significantly changes the response to the stimulation of a single neuron and a network of synaptically interacting neurons [33]. Two stable regimes for a single neuron with dendritic dynamics can coexist: the oscillatory regime, where the stimulation alters only the firing rate of the neuron, and the quiet regime, where the neuron stops firing completely [33].

Simple neural networks comprising only two neurons, one excitatory and the other inhibitory, may result into complex chaotic dynamics [34].

On the other hand, simple excitatory-inhibitory neural pairs (which form the building block of larger networks) subjected to external periodic stimulation may demonstrate the transition between various types of dynamics, depending upon the magnitude of the stimulus [35].

It is natural that more complex associative networks of neurons interconnected through an auto-correlation synaptic matrix may exhibit even more complex dynamics. However, the external stimulation of such network corresponding to a specific pattern may result into nearly periodic dynamics of the whole network [36].

Due to the high degree of connectivity within the brain, populations of neurons are constantly under the effect of external synaptic input. And although the firing rates of cortical background input is relatively low, studies have demonstrated its dramatic effect on the integrative properties of neurons in vivo, resulting in extreme variability of cortical responses. Therefore, taking into account the phase-independent external stimulation is justified and highly relevant for developing a single pulse control strategy for a network of dendritic neurons [37].

The main objective of this paper is to adapt a single pulse based control strategy for a dendritic neuron driven by external phase-independent stimulation. Similar control strategies have been used for a random network of dendritic neurons in [38] – but only weak periodic stimulation of the neurons was considered then. As mentioned previously, phase-independent stimulation may have a dramatic effect on the complexity of the dynamics of neurons. We will demonstrate that basin boundaries of coexisting states of the neuron with phase-independent stimulation (the oscillatory firing regime and the quiet regime) become highly complex and fractal. Therefore, the main novelty of this paper is twofold. First of all, we will develop weak phase-dependent control techniques which will be used to eliminate fractal boundaries between the coexisting states of the neuron. Secondly, we will develop and adapt single pulse based reliable control strategy which will be able to bring the neuron with phase-independent stimulation to the calm state.

2. The model of a neuron with dendritic dynamics

We will use the model of a neuron where the dynamics of the dendrite and the phase dynamics of the axon are described by the following differential equations [33]:

$$\dot{\psi}_i(t) = \alpha f(\phi_j(t)) - \gamma \psi_i + \beta \xi(t), \quad (1)$$

$$\dot{\phi}_i(t) = c\psi_i(t) + \omega_i, \quad (2)$$

where ψ_i is the dendritic current of the i th neuron, $f(\phi_j)$ is the synaptic input from the j th neuron, α is the synaptic strength, γ is the damping inherent in the dendrite, $\xi(t)$ is Gaussian white noise, statistically independent for each neuron, with zero mean and a standard deviation equal to one, β is the noise intensity, c is the scaling constant and ω is the driving force. The resulting governing differential equation reads:

$$\ddot{\phi}_i(t) = Af(\phi_j(t)) + \gamma(\omega_i - \dot{\phi}_j(t)) + c\beta\xi(t), \quad (3)$$

where $A = c \cdot \alpha$ and $\ddot{\phi}_i$ represents the dendritic currents. After substituting $m = 1/\gamma$; $\sqrt{2D} = c\beta/\gamma$, and introducing coupling and stimulation, the governing equation of the dendritic neuron takes the following form [33]:

$$m\ddot{\phi}_j = \omega_j - \dot{\phi}_j + W_j + S_j(t, \phi_j) + \sqrt{2D}\xi(t), \quad (4)$$

where m is the effective inertia of the oscillator; W_j is the dynamic coupling between the j th neuron and other neurons of the network and $S_j(t, \phi_j)$ is the phase-dependent stimulation of j th neuron [33]:

$$S_j(t, \phi_j) = A(t) \cos(\phi_j), \quad (5)$$

where $A(t)$ is a time dependent stimulation intensity function. The phase of a neuron is normalized to the interval between 0 and 1. The state of the neuron is described by the firing function [33]

$$\sigma(\phi) = \begin{cases} 1, & \text{if } \cos(2\pi \bmod(\phi, 1)) > c, \\ 0, & \text{otherwise;} \end{cases} \quad (6)$$

where the constant c determines when the phase of the neuron vanishes (modulus one), e.g., when it fires. We will assume that $c = 0.975$.

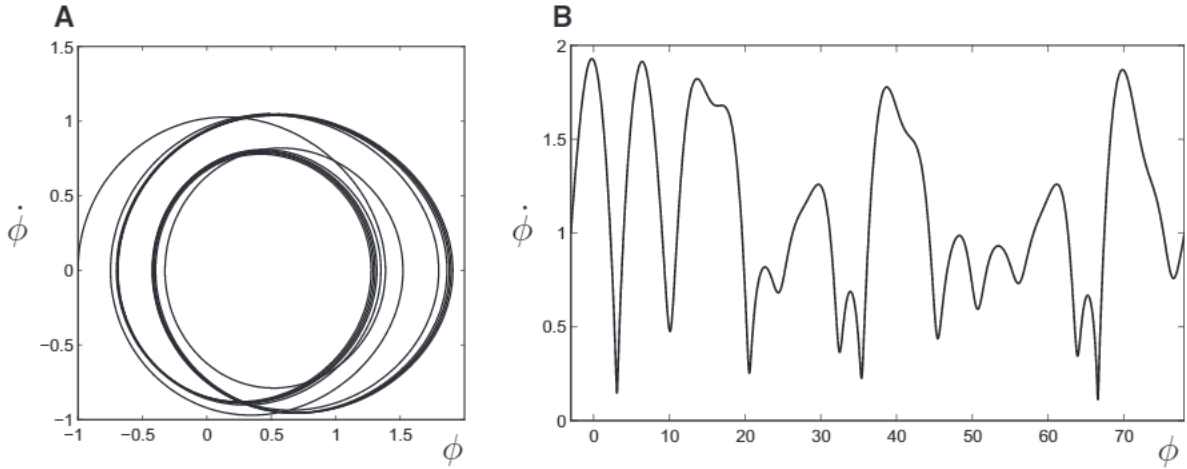


Fig. 1. The dynamics of a dendritic neuron driven by a phase-independent stimulation is sensitive to initial conditions – a bounded trajectory (A) coexists with an open phase trajectory (B). The dynamics of the neuron is given by $0.8\ddot{\phi} + 0.11\dot{\phi} + 0.3\sin(\phi) = 0.11 + 0.4\cos(\omega t)$; the trajectory stays in the bounded region of phase space when initial conditions are $\phi(0) = -1$ and $\dot{\phi}(0) = 0$ (part A). The phase space trajectory diverges to infinity when the initial conditions are $\phi(0) = -3$ and $\dot{\phi}(0) = 1$ (part B).

The dynamics of the phase of a single neuron under continuous excitation ($A(t) = a > 0$ constant) in absence of coupling and noise are described by the following differential equation:

$$m\ddot{\phi}_j = \omega - \dot{\phi}_j + a \cos(\phi_j); \quad (7)$$

The latter equation represents a nonlinear pendulum with constant drag moment ω . Thus, for $a < \omega$, only a stable limit cycle exists. At $a = \omega$ a saddle-node bifurcation occurs. For $a \leq \omega$ (7) has two types of fixed points: a stable focus point ($\phi = \arccos(-\omega/a)$; $\dot{\phi} = 0$) and an unstable saddle point ($\phi = -\arccos(-\omega/a)$; $\dot{\phi} = 0$). When the excitation intensity a reaches a critical value a_h , a homoclinic bifurcation occurs and the stable limit cycle is destroyed. The system exhibits bistability in the region $\omega < a < a_h$: two stable attractors (the fixed point and the limit cycle) coexist [33].

The bistability of the system corresponds to two different neural behaviors. The focus point corresponds to a quiet regime when the neuron does not fire. The limit corresponds to an oscillatory regime when the neuron fires periodically. This coexistence of attractors enables the control strategy based on short external pulses [38]. Initially the system oscillates in a limit cycle. A short pulse moves the system to the basin of attraction of a stable focus point. The neuron stays calm after transitional processes cease down [38].

3. The model of the dendritic neuron driven by a phase-independent stimulation

The model of a single neuron with dendritic dynamics comprises a phase-dependent periodic stimulation $S_j(t, \phi_j)$ [33]; the dynamics of networks of such neurons is well explored [33,39,40]. But it is well known that the stimulation of a neuron can also be phase-independent [41].

Therefore we will consider a model of the dendritic neuron described by the following equation:

$$m\ddot{\phi} + h\dot{\phi} + a \sin \phi = h + b \cos(\omega(t + t_0)) \quad (8)$$

This non-autonomous equation describes the phase dynamics of the dendritic neuron (in the absence of coupling and noise) which is additionally perturbed by an external phase-independent harmonic stimulation. Here, h represents the damping coefficient, a is the amplitude of the phase-dependent stimulation of the neuron, b is the amplitude of the phase-independent stimulation and ω is the external stimulation frequency (note the difference between h and ω). We fix the parameter values $m = 0.8$, $h = 0.11$, $a = 0.3$, $b = 0.4$, $\omega = 0.85$ in our computational simulations.

It can be noted that the solutions of (8) cannot be represented by a 2-dimensional phase plane diagram. Nevertheless, this model of the neuron exhibits different behavior for certain values of the amplitude b . A bounded attractor coexists with an open phase trajectory at $b = 0.4$ (Fig. 1A and B); the evolution of the system is dependent to initial conditions. Basins of attraction of these two distinct regimes are plotted in Fig. 2A: black dots denote initial conditions which result in a periodic bounded attractor; white dots mark initial conditions resulting in open trajectories.

In the analogy to the model of the dendritic neuron without external phase-independent stimulation we assume that a bounded trajectory corresponds to the quiet regime and the open trajectory represents the firing of the neuron. The state of the neuron will be described by the firing function $\sigma/\langle\phi\rangle$ (though several modifications compared to the standard model of the dendritic neuron are necessary). First we introduce a substitution $\varphi = \phi + \arccos(-\omega/a)$ which enables a correct classification of states along the whole axis $\dot{\phi} = 0$ by utilizing the modulo operation the divisor of which has been multiplied by 2π in order to avoid spikes that would

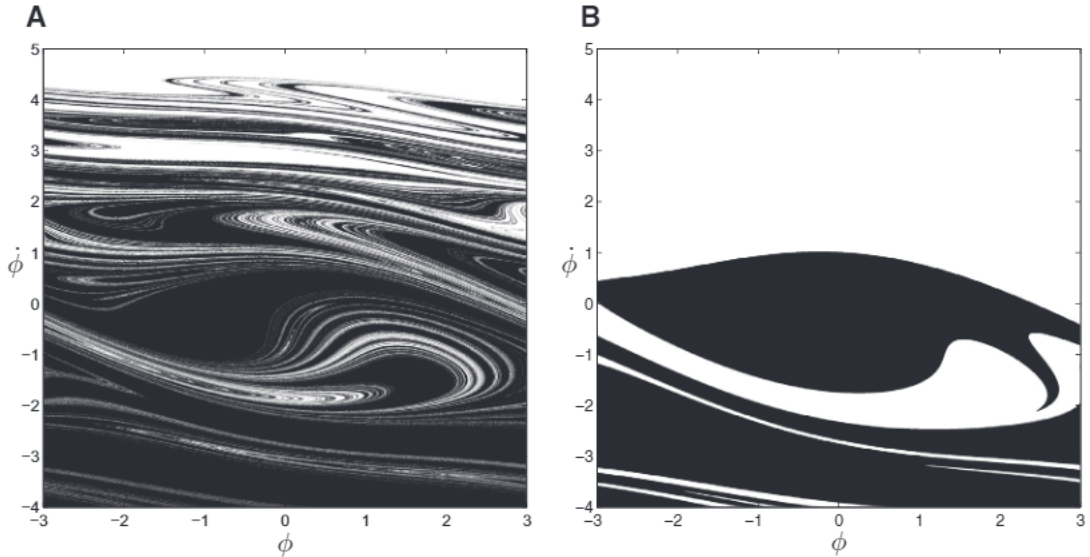


Fig. 2. Phase control can eliminate fractal basin boundaries. Basin boundaries of the neuron driven by a phase-independent stimulation $0.8\ddot{\phi} + 0.11\dot{\phi} + 0.3\sin(\phi) = 0.11 + 0.4\cos(0.85t)$ are fractal when no phase control is applied (part A). Black dots denote points which stay in the bounded region of the phase space while white dots denote points which escape to infinity. The basin of attraction is no longer fractal when phase control is applied: $0.8\ddot{\phi} + 0.11\dot{\phi} + (0.3 + a_0\cos(0.85t + \alpha))\sin(\phi) = 0.11 + 0.4\cos(0.85t)$ at $a_0 = 0.14$; $\alpha = 3\pi/2$ (part B).

otherwise be generated by a bounded phase plane trajectory. The reasoning for these modifications is based on the fact that bounded trajectories occur within the regions $(-\arccos(-\omega/a) + 2\pi k; -\arccos(-\omega/a) + 2\pi(k+1))$, $k \in \mathbb{Z}$ which follows from the dynamics of the dendritic neuron without external phase-independent stimulation (Fig. 5). Thus the modified function reads:

$$\sigma(\varphi) = \begin{cases} 1, & \text{if } \cos(2\pi \frac{\text{mod}(\varphi, d)}{d}) > c, \\ 0, & \text{otherwise;} \end{cases} \quad (9)$$

where the divisor $d = 2\pi$.

It can be observed that basin boundaries computed for (8) have fractal structure ($b = 0.4$; Fig. 2A). Fractal basin boundaries are typical phenomena in nonlinear dynamical systems. In general, a neural control strategy based on a short pulse fails to perform in a satisfactory way (Fig. 3). A short control pulse applied at the time t changes the conditions of the system instantaneously. The neuron calms down if the new state of the system corresponds to initial conditions of a bounded trajectory. But if the pulse is a little stronger the system evolves into an unbounded trajectory and the neuron continues firing (basin boundaries are fractal). Therefore, the control strategy based in a single strong pulse becomes unpredictable as the state of the neuron cannot be predicted due to the fractal nature of the basins of attraction. The pulse-based neuron control strategy could be applicable if only the fractal structure of the basin of attraction could be converted into a smooth boundary beforehand.

It is well known that nonlinear oscillators can be controlled by a weak phase control method [42]. Phase control has also been attempted in order to control chaos in the nonlinear pendulum, a similar system to our neuron model [43]. We introduce a harmonic perturbation of the parameter a which represents the amplitude of

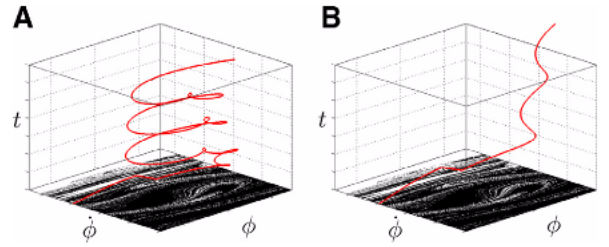


Fig. 3. Neuron dynamics control techniques based on an external pulse may fail when basin boundaries are fractal. The fractal basin of attraction for the equation $0.8\ddot{\phi} + 0.11\dot{\phi} + 0.3\sin(\phi) = 0.11 + 0.4\cos(0.85t)$ is shown at the bottom of both graphs (parts A and B). The control pulse (the strength of the pulse equals -19.45) brings the system to the basin of attraction of the bounded trajectory (neuron dynamics are represented by a thick red solid line). However, a slightly weaker pulse (the strength of the pulse equals -19.40) results in an unbounded trajectory. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

the phase-dependent stimulation of the system. In other words, though we cannot control the external phase-independent stimulation of the neuron, we still can control the phase-dependent stimulation. The model of the system then reads:

$$\begin{aligned} m\ddot{\phi} + h\dot{\phi} + (a + a_0\cos(\omega(t+t_0) + \alpha))\sin(\phi) \\ = h + b\cos(\omega(t+t_0)) \end{aligned} \quad (10)$$

where a_0 is the amplitude of the harmonic perturbation of the phase-dependent stimulation and α is the phase difference between the perturbation and the external phase-independent stimulation (note that the frequency of the internal perturbation is equal to the frequency of the external perturbation [42]). We set $a_0 = 0.14$ in our simulations.

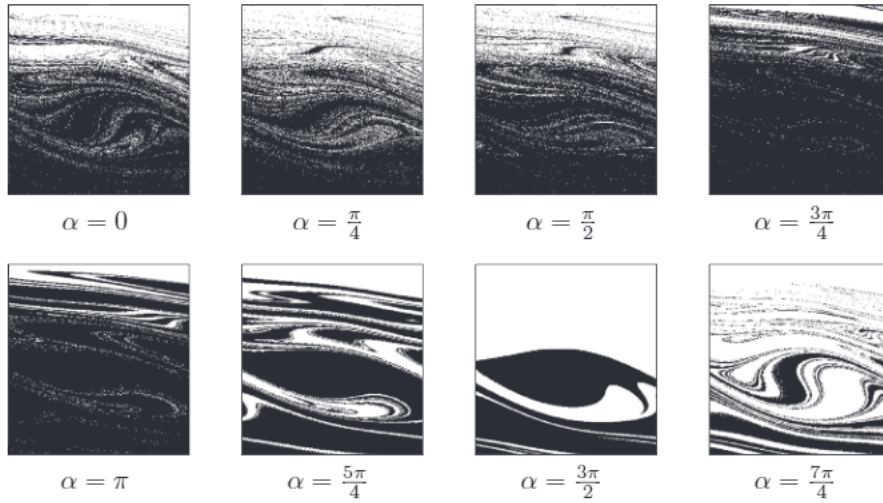


Fig. 4. The elimination of fractal basin boundaries by varying the parameter α (the phase difference between the perturbation and the external phase-independent stimulation). As described in [42], the best results are obtained by setting $\alpha = \frac{3\pi}{2}$; basin boundaries become deterministic at this value of α . The amplitude of the harmonic perturbation a_0 is set to 0.14 for all computational experiments.

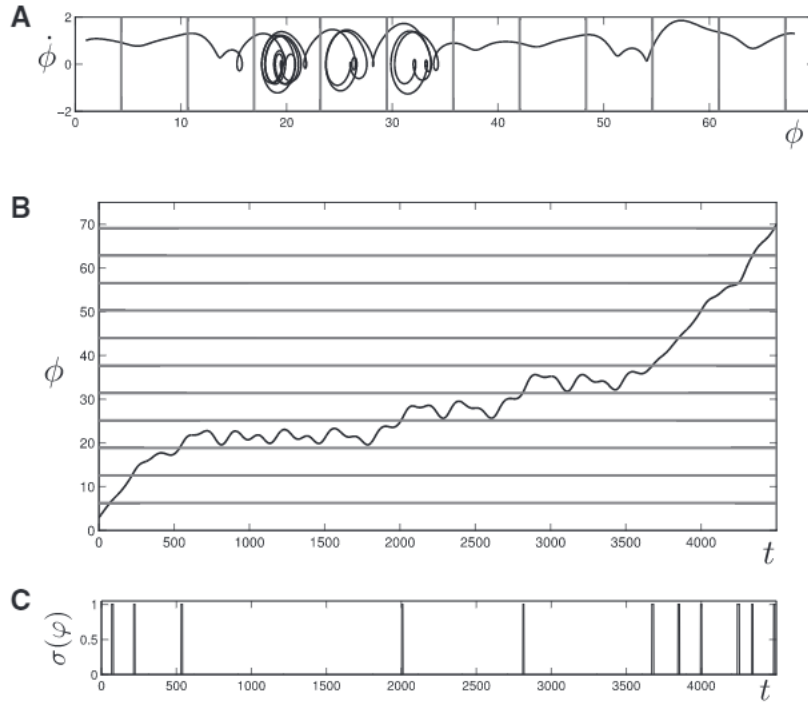


Fig. 5. Periodic bounded trajectories (corresponding to the quiet regime of the neuron) do not produce spikes when using the modified neuron firing function $\sigma(\varphi)$. The grid lines mark the values $-\arccos(-\omega/a) + 2\pi k$; $k \in \mathbb{Z}$. The trajectory in the phase plane is shown in (A); notice how the periodic behavior (which corresponds to the quiet regime) is confined to the interval $(-\arccos(-\omega/a) + 2\pi k; -\arccos(-\omega/a) + 2\pi(k+1))$, $k \in \mathbb{Z}$. The graph of $\phi(t)$ helps visualize the evolution of the system with time – pulses are only generated when values of ϕ cross the grid lines (B). The corresponding variation of the state on the neuron is illustrated in (C).

It is interesting to note that the basin of attraction of (10) has a complex structure. But fractal basin boundaries disappear at $\alpha = 3\pi/2$ (Fig. 2B and Fig. 4). The smoothness of the boundaries of the basin of attraction now can ensure that the control strategy based on a single control pulse can work well. Nevertheless, the modified model of the neuron represents a nonlinear

non-autonomous dynamical system. Thus, smooth basin boundaries can be produced by applying the weak phase control technique, but the particular shape of the smooth boundaries depends on the initial times t_0 . Thus, initial conditions corresponding to an open trajectory at time t_0 may correspond to a bounded trajectory at another time t_1 .

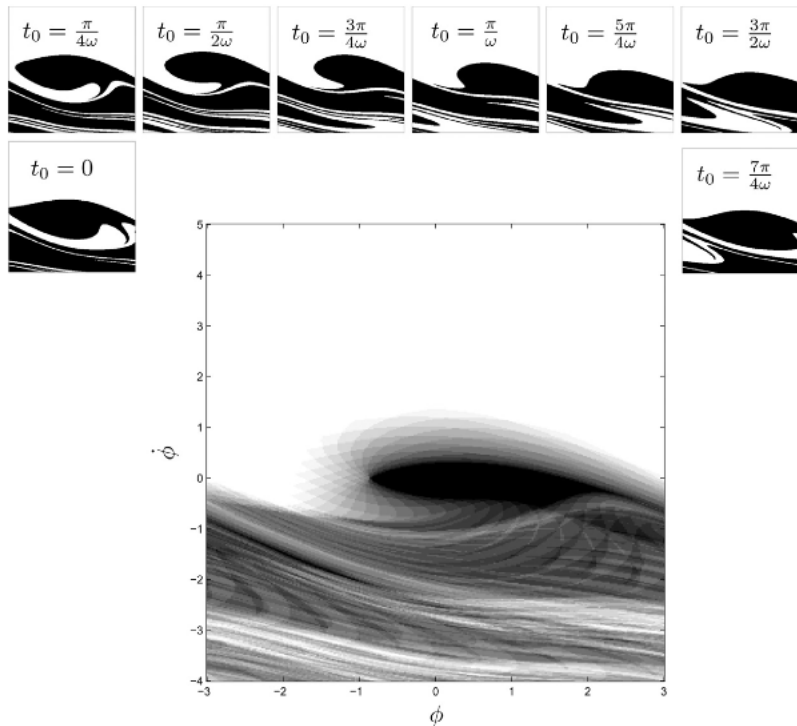


Fig. 6. The construction of a combined basin boundary for the neuron driven by phase-dependent stimulation. Small figures correspond to the basin boundaries of the system at different initial times t_0 . The large figure is an arithmetic mean of all individual basin boundaries (small figures) and presents a combined basin boundary. Black dots denote initial conditions of the bounded trajectories; white dots denote initial conditions of open trajectories for all initial times t_0 . Different grayscale levels correspond to the fraction of initial conditions $(\phi, \dot{\phi})$ of the bounded trajectories calculated at different initial times t_0 .

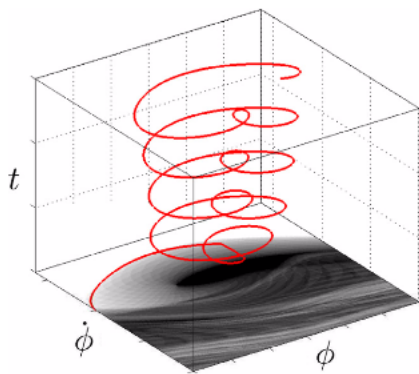


Fig. 7. The combined non-fractal basin of attraction for the neuron driven by an external phase-independent stimulation and harmonic perturbation of the controllable phase-dependent stimulation $0.8\dot{\phi} + 0.11\phi + (0.3 + a_0 \cos(0.85(t + t_0) + \frac{3\pi}{2})) \sin(\phi) = 0.11 + 0.4 \cos(0.85(t + t_0))$ is shown at the bottom of the 3-dimensional graph. The control pulse calms the neuron (the trajectory of the system is illustrated by a thick red solid line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

Therefore, a combined basin of attraction is constructed for every time $0 \leq t_0 \leq 2\pi/\omega$ (Fig. 6). A number of smoothed basin boundaries are constructed for several discrete values of t_0 . It is important to note that the phase shift of the internal perturbation $\alpha = 3\pi/2$ is fixed for all different initial time t_0 (this is the optimal phase shift

when the annihilation of the fractal boundaries is considered). Next, we average all smoothed basin boundaries. White regions represent initial condition of open trajectories for all initial times; black regions denote initial conditions of bounded trajectories for all initial times; and the grey regions represent initial conditions which fall into the attractor for some values of t_0 and diverge to infinity at others. Now, a predictable control strategy based on a single control pulse can be considered.

The system moves from an open to a bounded trajectory after a single pulse is applied if the strength of the pulse is sufficient (Fig. 7).

4. Conclusions

Control strategies based on weak phase synchronization are developed for a dendritic neuron driven by external phase-independent stimulation. The control strategy is based on a single control pulse which pushes the system into the basin of attraction of the stable bounded trajectory. Unfortunately, basin boundaries of coexisting states (the oscillatory synchronized regime and the quite regime) are fractal. The control method becomes unreliable due to the simple reason that the evolution of the neuron (after the impulse is applied) becomes unpredictable. Weak phase synchronization helps to overcome the mentioned difficulties. Fractal basin boundaries between the firing and the calm state of the neuron evolve into a smooth manifold

when weak phase synchronization is turned on. Then, the control strategy based on a single control pulse works well and the dynamics of the dendritic neuron evolves into a calm state. So far, we have conducted the analysis of a single neuron, however, the application of these ideas to the control of a neural network is a definite object of future research.

Acknowledgments

Financial support from the Lithuanian Science Council under Project no. MIP-078/2015 is acknowledged.

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