

# A technique for the reconstruction of a map of continuous curves from interference fringes

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## ABSTRACT

The well-known phase-shifting approach for three-dimensional surface measurement uses multiple fringe patterns along with the phase-shifting algorithm to obtain 3-D profiles with high accuracy though this approach is not applicable for dynamic object measurement techniques such as time-averaged holography and in cases when only a single interference fringe pattern is available. In this case the fringe tracing method can be used that is based on localization of centers of interference fringes. We propose a technique for the reconstruction of the contour map from fringe patterns which comprises standard image processing techniques and a scheme for reconstruction of the map of continuous curves from the binary matrix of pixels representing fringe centers. The approach of image division into grid cells is taken and such problems as derivation of approximate line equations in each cell using Hough transformation, grouping contacting cells into curves and interpolation between curves with fractures are solved. The functionality of this approach is demonstrated for a demanding optical image containing fractures and noise.

**Keywords:** Interference fringes, Fringe pattern processing, Metrology

## 1. INTRODUCTION

Strain measurement and control tools are primary technology drivers in such industries as MEMS (micro-electro-mechanical systems) fabrication and high-precision machine tools manufacturing.<sup>1-4</sup> Reliable interpretation of experimental measurement results allows minimization of the general uncertainty budget. Particularly that is important in situations when complex surfaces of the investigated systems and noisy background does not allow a reliable interpretation of the optical measurement results.

One of the commonly used methods for reconstruction of 3D map of digital holography is phase shifting technique.<sup>5-7</sup> Phase shifting technique has advantage over classical analysis of static interferograms when centerlines of a fringes are identified as this method is prone to errors. Phase shifting technique uses different approach of collecting intensity data from every point of interferograms which then are analyzed point by point with the reference optical surface shifted by a precise fraction of a wavelength between each exposure.<sup>8</sup>

Nevertheless this method is not always suitable for example when we have only one interferogram.<sup>9</sup> The other case is when we have an interferogram produced by time-averaged holography method. In this article we are using a method when interpretation of the interference fringes can be separated into two basic blocks – the identification of centerlines of the projected fringes and the reconstruction of maps of the measured physical quantity (strain, amplitude of deformation, etc.) . And while the second task can be taken as an almost standard feature of any contour-based 3D plotting software package, the computational derivation of the contour maps is still very much dependent on particular implementations. Applicability of the digital holography method for more

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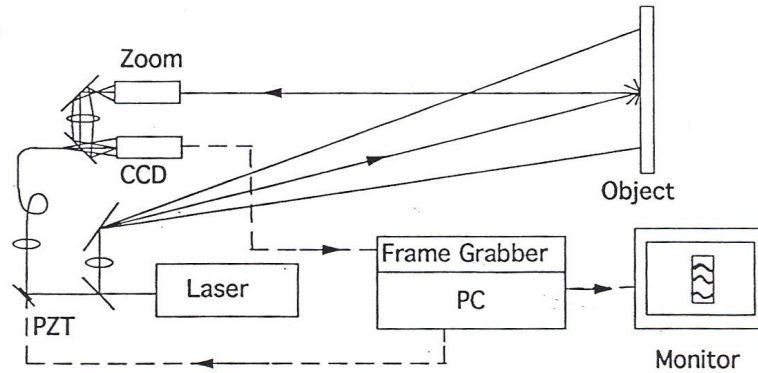


Figure 1. Optical setup for PRISMA system.

complex structures in more demanding environments requires reliable computational tools for the interpretation of experimental images. The reconstruction of the 2D map of interference fringes centerlines is one of the main factors ensuring the accuracy of the interpretation of optical measurement results. The main objective of this paper is to propose a general scheme for the construction of the centerline contour map from an experimental optical image for holographic interferograms.

The article is structured as follows. Details about an experimental setup is given in Section 2. The method of identification of continuous curves is given in Section 3. The results for reconstruction of continuous curves is discussed in Section 4. Finally, concluding remarks are given in Section 5.

## 2. THE EXPERIMENTAL SETUP

At this moment the laser interference holographic analysis method is most suitable for the analysis of such systems. The methods of holographic interferometry allow to reconstruct much more information about the deformable surface comparing with other experimental methods.<sup>10</sup> In this paper the presented PRISM's technology uses real-time, 3D, full field-of view surface measurements to eliminate point-by-point data gathering. Holograms are used to measure the surface of any component. Available resolution enables monitoring shape changes that are smaller than 20 nanometers for superb accuracy and resolution what is important for equipment manufactures that are dissatisfied with the cumbersome and costly laser scanning techniques that digitize discrete points on a surface over a much longer time frame. PRISM offers a new real-time digital holographic technique that provides results within minutes at a fraction of the cost. The main benefits of PRISM system are the following:

- Fast measurements: typical measurement and analysis in less than 5 minutes
- High-resolution: monitor shape changes much less than 20 nanometers
- Customizable: affordable solution designed specifically for your application
- Accurate: more data with full field of view, real-time surface measurements
- Non-contact: only requires direct visual path to part
- User friendly: Windows based software

The tests used the PRISMA system layout shown in Figure 1. The PRISMA system shown in Figure 1 is a two beam speckle pattern interferometer. The object beam is the laser beam directed at the object. The reference beam is the other beam going directly to the camera. Laser light is scattered from the object and collected by the camera lens, which also images the object onto the CCD camera sensors. The reference beam goes directly to the camera, usually in an optical fiber, where it overlaps the image of the object shape changes that occur

between a reference and a stressed state of the object produce fringes on top of the image of the object which is displayed on the TV monitor. An experimental image is shown in Figure 2.

Successful usage of the PRISM system depends on the quality of the interference fringes, because any damages in the system of interference fringes may lead to unwanted effects, and it could be problematic to process data by a computer in order to build full field displacements of the surface of investigated objects. Distortions in the system of interference fringes may occur because of reflecting properties of surface of investigated object, quality of the optical element used in the optical setup, etc.. Time average holographic interferometry method for the recording of deformable surfaces of vibrating objects also can induce distortions and discontinuities in the system of interference fringes. Experimental investigations were based on time average laser ESPI holography concentrating on the dynamics of the rectangular piezoceramic plate. ESPI holographic interferogram is presented in Figure 2.

### 3. THE METHOD OF IDENTIFICATION OF CONTINUOUS CURVES

The proposed technique comprises 4 main steps: thresholding, identification of centerlines, identification of curves and joining of non-continuous curves.

#### 3.1 Thresholding

The first step towards the identification of centerlines is the construction of a binary representation of the optical experimental image. The quality of the resulting binary image is crucial for the remaining steps. Various thresholding techniques can be exploited for the construction of the binary representation of the original experimental image (though not all of them may produce a desired result in different conditions).<sup>11</sup> Thresholding may be viewed as an operation that involves tests against a function  $T$  and a thresholded image can be defined as:

$$g(x, y) = \begin{cases} 1 & f(x, y) < T(x, y, f(x, y), p(x, y)) \\ 0 & \text{otherwise} \end{cases}$$

where  $f(x, y)$  is the gray level of point  $(x, y)$  and  $p(x, y)$  denotes some local property of this point.<sup>12</sup> Various local and global thresholding methods were tested but the results showed that application of global thresholding methods (such as histogram shape-based methods) does not give desired outcome. Local thresholding based on average intensity value of neighboring pixels surrounding the current pixel gives good results.

The structure of the original optimal image suggests that initial smoothing before the thresholding procedure may help to gain better representation of fringes in the thresholded image.

#### 3.2 Identification of Centerlines

Two alternative techniques for the identification of centerlines from the thresholded experimental image are often used. The first method is based on the identification of centerlines using the mid-point method and the second technique is based on mathematical morphological operations. The first method is based on the algorithm proposed in<sup>13</sup> and later investigated in.<sup>14</sup> It uses vertical and horizontal scan lines to determine mid-points of each dark and bright fringe. Nevertheless, direct application of this algorithm produces poor results due to large number of fractures in the pattern of centerlines and loose spurs in areas between centerlines.

Mathematical morphology<sup>12</sup> is a tool for extracting image components that are useful in representation and description of regions or shapes such as boundaries of a region, skeletons or other patterns. Various morphological operations like dilation, erosion, opening, closing, thinning, thickening and others have been developed and are successfully applied in different areas of pattern recognition science and technology.<sup>15</sup> In this paper we will use the thinning operation based on the hit-or-miss algorithm since simple erosion operation would destruct thinner parts of moiré fringes in the thresholded image. The thinning operation is defined through the erosion and the hit-or-miss operations.<sup>12</sup> The execution of the thinning process might produce undesired parasitic components (spurs) around actual centerlines of interference fringes. These spurs are caused during erosion by non-uniformities in objects and may be removed by performing the pruning operation.<sup>12</sup>



Figure 2. The optical experimental image.

### 3.3 A Method for the Identification of Curves

A proposed method for joining broken centerlines requires identification of continuous intervals of curves in produced digital image. A method for grouping sets of adjacent pixels into curves comprises following steps of grouping individual pixels into cells, derivation of approximate line equation in each cell and grouping contacting cells into curves.

The selection of the size  $w_g$  of the cell should be related to the density of moiré fringes in the original optical image. The main requirement is that there should be only one centerline passing through each cell. On the other hand a larger cell size increases the performance of the proposed method.

In order to derive approximate line equation in each cell Radon<sup>16</sup> or Hough<sup>17</sup> transforms can be exploited and it is probably the most reliable solution for robust identification of a line in a cell. Both transforms can be effectively used for the detection of simple shapes such as lines, circles, ellipses by making a transformation from the image space to the parameter space. A line in plane can be defined by:

$$x \cos(\theta) + y \sin(\theta) = r$$

where  $r$  is the distance between the line and the origin (the bottom-left corner of the rectangular cell);  $\theta$  is the angle of the vector from the origin to this closest point of the line. The main idea behind Hough transform is based on the fact that points in the image space correspond to sinusoids in the parameter space. If an array of points in the image space does form a line, then the intersection of sinusoids creates a point in parameter space (parameters of the line in the image space can be deduced from the coordinates of that point). To gain better performance a variant of Hough transform - Randomized Hough transform<sup>18</sup> was selected and then applied to every cell of the thinned image. A new matrix of vectors comprising basic line parameters (the angle  $\theta$  and the location of the line mid-point  $x$  and  $y$  in each grid cell) is constructed:

$$G = \{(\theta_{i,j}, x_{i,j}, y_{i,j} \mid i \in [1, m_g], j \in [1, n_g])\}$$

where  $m_g \times n_g$  is the number of cells in the vertical and the horizontal directions.

Grouping contacting cells into curves is performed as follows:

- (i) Proceed with cells from left to right and from top to bottom starting at cell  $G_{1,1}$ .
- (ii) Check if there are cells with an assigned line in the immediate surroundings of  $G_{i,j}$ .
- (iii) Select a nearest non-assigned cell with the derived line equation  $G_{k,j}$  to the current cell  $G_{i,j}$ .  
Assign to the set of cells belonging to the same curve  $C_i$ .

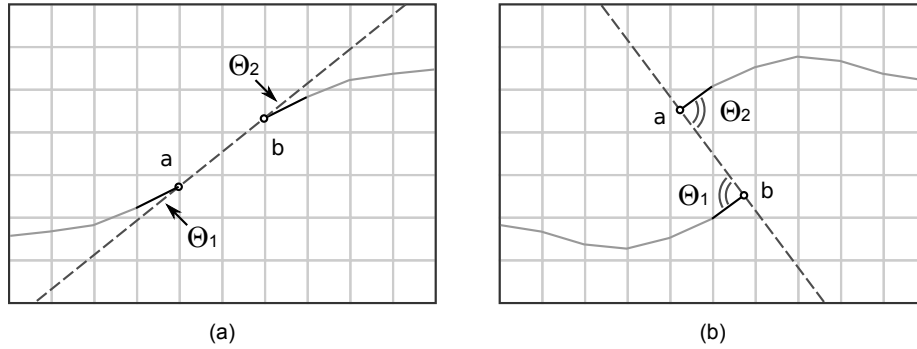


Figure 3. A schematic diagram illustrating a favorable connection between two curves is demonstrated in (a) in which the criterion  $a_0$  would be sufficient for the detection of similar angles between lines in the edge cells. A schematic diagram illustrating an unfavorable connection between two curves is demonstrated in (b) where criterion  $a_1$  and  $a_2$  eliminates a possible connection between the edge cells.

- (iv) Repeat steps (ii - iii) with cell  $G_{k,j}$ .
- (v) Return to cell  $G_{i,j}$  and continue with the next cell in a row (column); go to step (ii).

It is possible that curves may have fractures larger than one cell due to defects in the initial image or loss of information during the previously described steps. An adaptive method for grouping continuous curves is given in the following section.

### 3.4 A Method for Joining Non-Continuous Curves

A set of curves is already identified (each curve is associated to a set of cells). We define criteria that are used to decide if two separate curves could be joined:

$$d = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \quad a_1 = |\theta_i - \theta_{i,j}|, \quad a_2 = |\theta_j - \theta_{i,j}|$$

where  $d_{i,j}$  is the distance between the edge cells of the curve,  $(x_i, y_i)$  and  $(x_j, y_j)$  are all possible pairs of center points of edge cells of the identified curves,  $\theta_{i,j}$  is the angle of the line going through points  $(x_i, y_i)$  and  $(x_j, y_j)$ . The other criterion is orientation between edge cells of curves:

$$a_0 = |\theta_i - \theta_j|$$

though, a number of computational experiments have shown that there could be situations where that criterion does not work as expected, thus an alternative criteria  $a_1$  and  $a_2$  are used.

We also define the applicable value of the parameter  $J_{i,j}$  which is used when there are more than one curve holding against defined criterion constraints:

$$J = \frac{1}{d + \alpha(a_1 + a_2)}$$

where parameter  $\alpha$  is used as weight and allows to emphasize the importance of the distance between the edges or the variation of angles at the edges. The defined curve joining criteria are incorporated into the proposed algorithm for curve joining:

- (i) Select a curve  $C_i$  from the set  $C$ .
- (ii) Compute criteria  $d$ ,  $a_1$  and  $a_2$  between edges of curves  $C_i$  and  $C_j$ .
- (iii) Merge all cells of the curve  $C_j$  with  $C_i$  for such  $i$  and  $j$  minimizing the value of  $J$ .  
Repeat steps (ii-iii) until there are no more favorable connections.
- (iv) Proceed with the curve and repeat steps (i-iii).

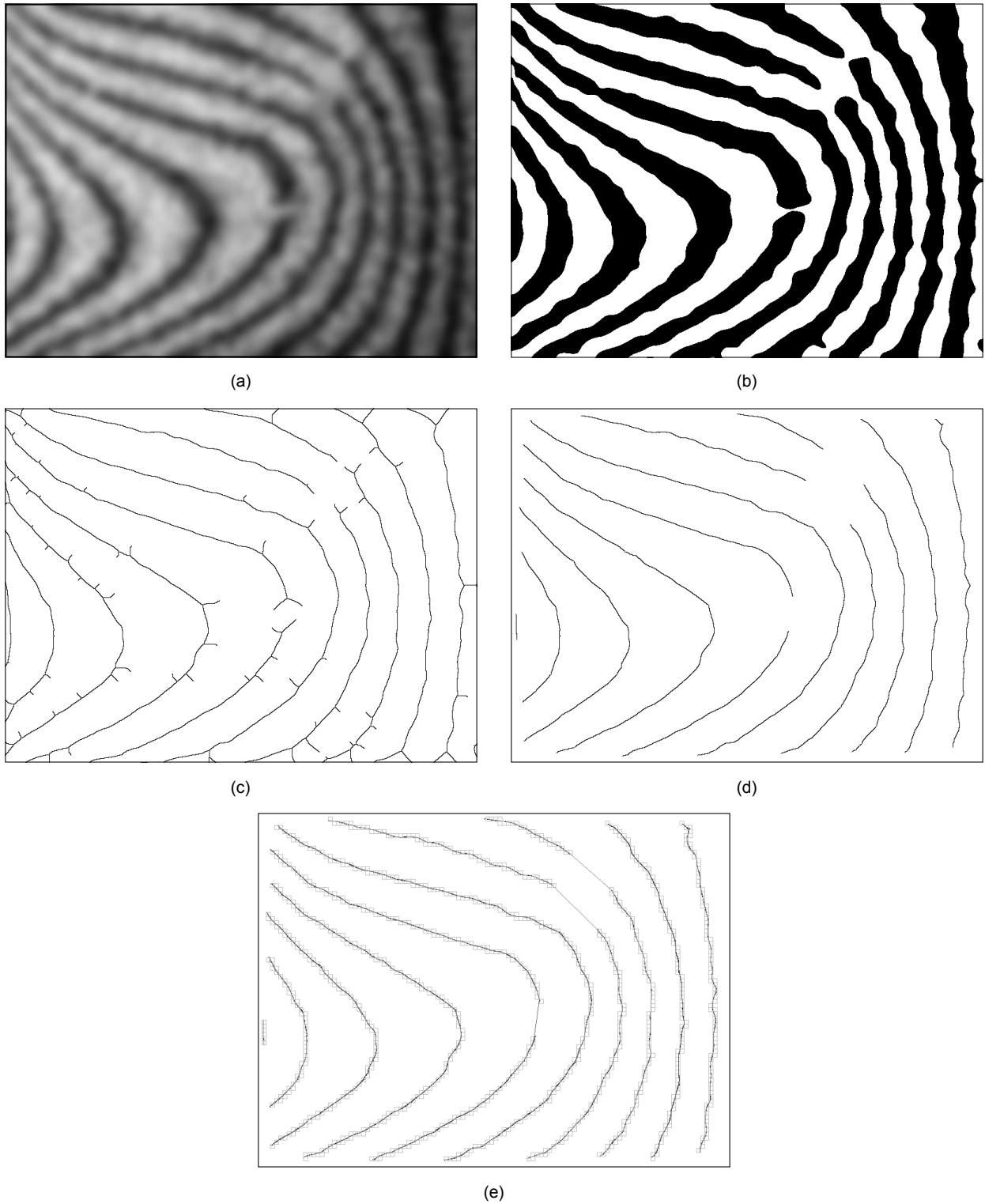


Figure 4. Gaussian smoothing operation is applied to original image in (a); local thresholding algorithm is performed in (b); mathematical morphological operations are performed in (c); spurs are removed in (d); final result after identifying individual curves and joining them as described in Sections 3.3 and 3.4.

## 4. THE RESULTS FOR RECONSTRUCTION OF CONTINUOUS CURVES

Gaussian smoothing operation is applied to the original image in Figure 4 (a). This allows the removal of noise and achieving a smooth thresholded interference fringes that are visible in Figure 4 (b). A local thresholding method is used because of an uneven intensity in original and smoothed images: one can notice that a right side of image is darker than a left side. Application of mathematical morphological operations can be seen in Figure 4 (c) and (d). Firstly a thinning operations is performed and later spurs removing operation is performed in order to get image Figure 4 (d). A smooth and noise-free pattern of centerlines is produced in the result of the described digital image processing operations. Unfortunately, a number of intermittent broken centerlines can still be visible. Some of these defects are caused by optical defects in the original image. Appropriate joining of broken centerlines could considerably improve the quality of interpretation of the optical experimental image. The results of smooth and noise-free pattern of centerlines is shown in Figure 4. Unfortunately, a number of intermittent broken centerlines can still be visible in because of optical defects in the original image. Final result after identifying individual curves and joining them as described in Sections 3.3 and 3.4 can be seen in 4 (e) - the map of continuous curves is reconstructed and separated curves are joined.

## 5. CONCLUDING REMARKS

A technique for reconstruction of the map of continuous curves from interference fringes of optical experimental images is proposed. This scheme is based on standard digital image processing techniques and a proposed line tracing methods which make this scheme robust to noise and various distortions in the original optical image. The main objective of this article is to propose a reliable scheme for the construction of 2D map of fringe centerlines, since that in the main source of errors and uncertainties in the quantitative interpretation of digital holography images. Robust identification of interference fringe centerlines is not only important in digital and time-averaged holography, but can also be applied for other methods like shadow and projection moiré which also generates interference fringes. It is important for such applications where non-uniform distribution the observation area, complex surfaces of the investigated systems and noisy backgrounds are common and do not allow a reliable interpretation of optical measurement results - what is especially important in optical MEMS analysis.

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